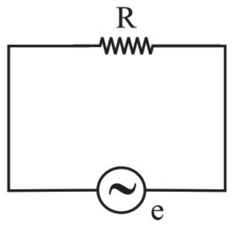
AC CIRCUITS

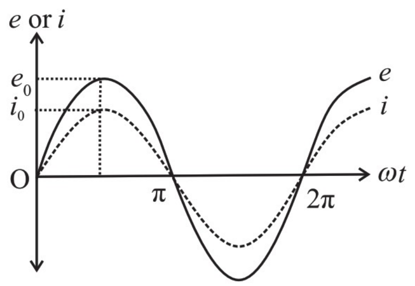
RESISTIVE CIRCUIT



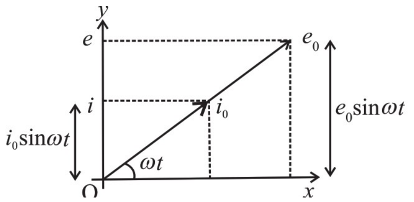
$e = e_o \sin(\omega t)$

$i = \frac{e}{R} = \frac{e_o}{R}\sin(\omega t) = i_o \sin(\omega t)$

Waveform:



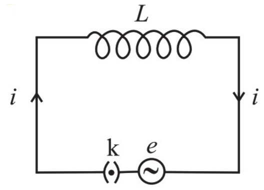
Phasor Diagram:



Power P $= e_{rms} \times i_{rms} \cos\phi$

$= \frac{e_o\, i_o}{2} \cos 0^o = \frac{e_o\, i_o}{2}$

INDUCTIVE CIRCUIT



$Induced\ emf\ e_b = -L\frac{di}{dt}$

$Applied\ emf\ e\ should\ be\ equal$

$and\ opposite\ e = -e_b = L\frac{di}{dt}$

$di = \frac{e}{L}dt.\ \ Integrating,$

$\int di = \int \frac{e}{L} dt$

$i = \frac{1}{L}\int e_o \sin(\omega t)\, dt$

$= \frac{e_o}{L}\left[\frac{-\cos(\omega t)}{\omega}\right]$

$= -\frac{e_o}{\omega L}\cos(\omega t) = -\frac{e_o}{X_L}\cos(\omega t)$

$i = -\frac{e_o}{X_L}\sin\left(\frac{\pi}{2} - \omega t\right)$

$i = \frac{e_o}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$

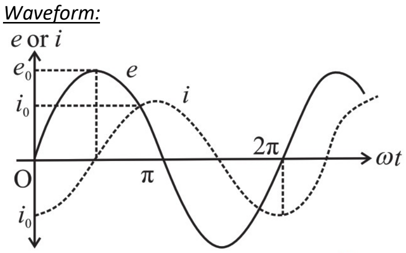
$Thus, current\ lags\ voltage\ by \frac{\pi}{2}$

$where\ X_L = \omega L = 2\pi f L$

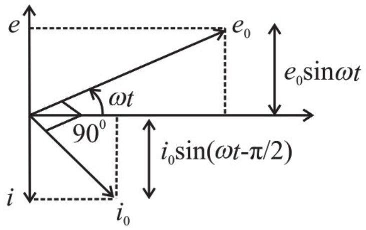
$is\ the\ reactance\ of\ the\ inductor$

$S.I.Unit\ \Omega$

Waveform:

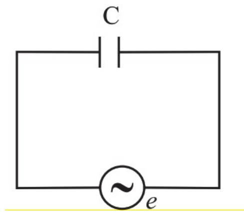


Phasor:



P $= e_{rms} \times i_{rms} \cos\frac{\pi}{2} = 0$

CAPACITIVE CIRCUIT



$i = \frac{dq}{dt} = \frac{d(C.e)}{dt}$

$i = \frac{d}{dt}(C.e_o \sin(\omega t))$

$i = Ce_o \frac{d}{dt}\sin(\omega t)$

$i = Ce_o \cos(\omega t).\omega$

$i = \frac{e_o}{\frac{1}{\omega C}}\cos(\omega t) = \frac{e_o}{X_C}\cos(\omega t)$

$i = \frac{e_o}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right)$

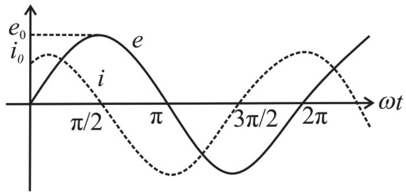
$Thus, current\ leads\ voltage\ by \frac{\pi}{2}$

$where\ X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

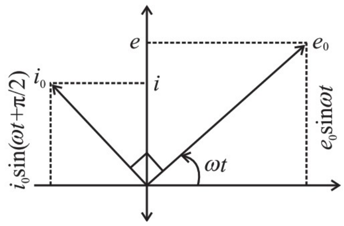
$is\ the\ reactance\ of\ capacitor$

$S.I.Unit\ \Omega$

Waveform:



Phasor :



P $= e_{rms} \times i_{rms} \cos\frac{\pi}{2} = 0$

NOTE:

$for\ DC, f = 0,$

$thus\ X_L = 0\ and\ X_C = \infty$

$Thus, coil\ or\ inductor\ act\ as\ short$

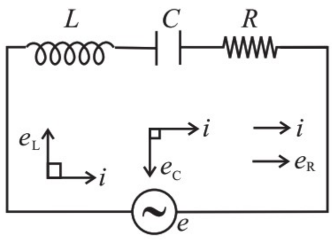
$circuit\ for\ DC\ and\ capacitor\ blocks\ DC$

As f increases, X_L increases
and X_C decreases

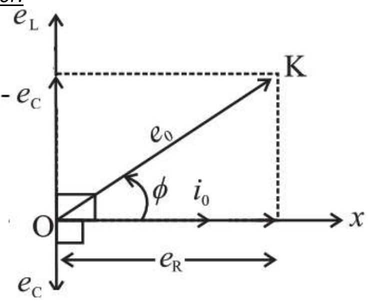
Telephone: 9820246760 / 9769479368 • Websites: navlakhi.education
Email: abhishek@navlakhi.com –or– kunal@navlakhi.com

Navlakhi®

SERIES LCR CIRCUIT



Phasor:

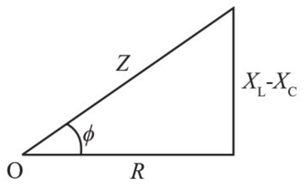


$e = \sqrt{e_R{}^2 + (e_L - e_C)^2}$

$iZ = \sqrt{(iR)^2 + (iX_L - iX_C)^2}$

$\boldsymbol{Z = \sqrt{R^2 + (X_L - X_C)^2}}$

Impedance triangle



NOTE:

$\tan\phi = \dfrac{X_L - X_C}{R}$

$(i) If\ X_L > X_C,$
$\tan\phi\ is\ positive\ and$

$circuit\ will\ be\ inductive,$
$voltage\ will\ lead\ current\ by\ \phi$

$(ii) If\ X_L < X_C,$
$\tan\phi\ is\ negative\ and$

$circuit\ will\ be\ capacitive,$
$voltage\ will\ lag\ current\ by\ \phi$

$(iii)\ If\ X_L = X_C,$
$\tan\phi = 0\ , circuit\ is\ resistive$

$Z_{min} = R\ and$

$current\ is\ maximum\ i_o = \dfrac{e_o}{R}$

SERIES RESONANCE (Acceptor Circuit)

$If\ X_L = X_C,$

$\omega L = \dfrac{1}{\omega C}, \qquad \omega^2 = \dfrac{1}{LC}$

$\omega = \dfrac{1}{\sqrt{LC}}$

$thus, Resonant\ frequency$

$\mathbf{f = \dfrac{1}{2\pi\sqrt{LC}}}$

$At\ resonance,$

$\tan\phi = 0\ , circuit\ is\ resistive$

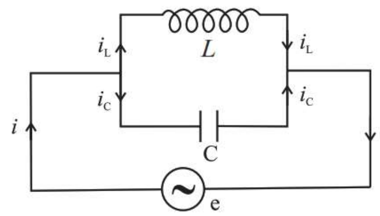
$Z_{min} = R\ and$

$current\ is\ maximum\ i_o = \dfrac{e_o}{R}$

$P = e_{rms}\ .i_{rms}.cos\phi$

$P = \dfrac{e_o}{\sqrt{2}}.\dfrac{i_o}{\sqrt{2}}.cos0 = \dfrac{e_o.i_o}{2}$

PARALLEL RESONANCE (Rejector Circuit)



$If\ X_L = X_C,$

$\omega L = \dfrac{1}{\omega C}, \qquad \omega^2 = \dfrac{1}{LC}$

$\omega = \dfrac{1}{\sqrt{LC}}$

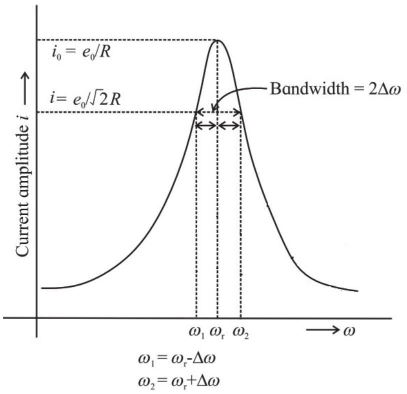
$thus, Resonant\ frequency$

$\mathbf{f = \dfrac{1}{2\pi\sqrt{LC}}}$

$At\ resonance, i\ is\ minimum$
$Z\ is\ maximum$

Q- FACTOR/QUALITY FACTOR

Q factor of a series resonant circuit is defined as the ratio of resonant frequency to the difference in two frequencies taken on both sides of resonant frequency such that at each of these frequencies the current amplitude is 1/√2 times the value at resonant frequency.



$Q = \dfrac{Resonant\ frequency}{Bandwidth} = \dfrac{\omega_r}{2\Delta\omega}$

Definition:

Peak value: Peak value of an alternating current (or emf) is the maximum value of the current (or emf) in either direction.

Average / Mean value:

Average value or mean value of one HALF cycle can be found as follows:

$Average\ or\ mean\ value\ \ of\ current$

$= \dfrac{\int_0^\pi i_o\ sin\theta\ d\theta}{\int_0^\pi d\theta} = \dfrac{i_o \int_0^\pi\ sin\theta\ d\theta}{[\theta]_0^\pi}$

$= \dfrac{i_o[-cos\theta]_0^\pi}{\pi - 0} = \dfrac{i_o[-cos\pi - (-cos0)]}{\pi}$

$= \dfrac{\mathbf{2i_o}}{\boldsymbol{\pi}} = \mathbf{0.637i_o}$

$\boldsymbol{Similarly, mean\ or\ avg\ value\ of\ voltage}$

$\boldsymbol{over\ one\ half\ cycle} = \dfrac{\mathbf{2e_o}}{\boldsymbol{\pi}} = \mathbf{0.637e_o}$

NOTE: Average value over a full cycle of sin or cos wave is zero.

Effective value / Virtual Value / RMS value:

Consider an AC current of peak value i_0 passing through a resistor R and producing heat H in time t. Then, the steady value of current, which produces the same heat in a resistor (R) in same time t is called as effective/virtual/RMS value of current.
RMS value can be found as follows:

$RMS\ Value\ of\ current = \sqrt{\dfrac{\int_0^{2\pi} i_o{}^2\ sin^2\theta\ d\theta}{2\pi}}$

$= \sqrt{\dfrac{i_o{}^2 \int_0^{2\pi}\ sin^2\theta\ d\theta}{2\pi}}$

$= \sqrt{\dfrac{i_o{}^2}{2\pi} \int_0^{2\pi} \left(\dfrac{1 - cos2\theta}{2}\right) d\theta}$

$= \sqrt{\dfrac{i_o{}^2}{4\pi} \left[\theta - \dfrac{sin2\theta}{2}\right]_0^{2\pi}}$

$= \sqrt{\dfrac{i_o{}^2}{4\pi}.2\pi} = \dfrac{i_o}{\sqrt{2}} = \mathbf{0.707}\ i_o$

$\boldsymbol{Similarly,}$

$\boldsymbol{RMS\ voltage} = \dfrac{\boldsymbol{e_o}}{\sqrt{\mathbf{2}}} = \mathbf{0.707}\ \boldsymbol{e_o}$